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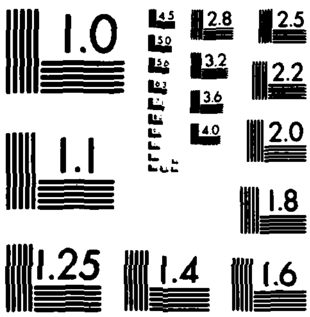

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LINCOLN LABORATORY

THE ML METHOD FOR FREQUENCY ESTIMATION OF REAL SINUSOIDS IN NOISE

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ABSTRACT

The DFT-based maximum likelihood method (MLM) for frequency estimation of complex sinusoids as proposed in [1,2] is extended to treat the case of real data. Improvements on estimation precision and computation efficiency are obtained by imposing an equal-frequency constraint on the pair of complex sinusoids which corresponds to a real sinusoid and by utilizing a single-step interpolation in conjunction with a coarse finite search over the DFT data for the maximum of the likelihood functon. Simulation results demonstrate these improvements.

CONTENTS

I. INTRODUCTION

The maximum-likelihood (ML) method for frequency estimation of sinusoids in noise was first proposed in [1] and extended in [2,3]. It has been demonstrated that this method is often an efficient estimator and superior to some other approaches [1,3]. In [1], a single-tone complex sinusoid in the discrete form was treated. The frequency of the sinusoid was identified as the one which maximizes the magnitude of the Fourier transform of the discrete signal. It was obtained by first searching coarsely over the DFT spectrum and then refining the estimate with the cosine iteration algorithm. This method was extended to handle multiple tones in [2] where the coarse search was performed in a frequency space with dimension equal to the number of complex sinusoids.

The ML method for the complex data can be used in two ways to treat real data without any modification. One way is to regard each real sinusoid as a pair of complex ones with frequencies equal in magnitude and opposite in sign. In doing so, the dimension of the search space is thus doubled and, as a consequence, the computation cost would increase significantly. Furthermore, the estimation precision would also degrade because the pair of frequency estimates would not be necessary equal to each other in magnitude, especially in the presence of noise or in the case of very low frequency

$(\omega \to 0)$.

The other way is to first generate an "analytic" (complex-valued) signal for the given real-valued signal using the Hilbert transform [4]. The difficulty with this approach is that, for a short discrete signal, the Hilbert transform could not produce a well-approximated complex sinusoid from each real one and it would distort the noise structure.

In this paper we present a refined ML method for frequency estimation of real sinusoids in which the dimension of search space is kept as the number of real sinusoids and the fine search is carried out using a single-step interpolation which requires little extra computation. Simulation results show that this refinement can improve the estimation precision and computation efficiency.

II. THE METHOD

A. Derivation

We assume that the observed signal $\{y_n, n=0,..N-1\}$ contains M real sinusoids corrupted by additive noise w_n.

$$y_n = \sum_{m=1}^{M} a_m \sin(\omega_m n+\theta_m) + w_n \quad n=0,1,..,N-1 \tag{1}$$

where the real positive amplitudes $\{a_m\}$, the phases $\{\theta_m\}$ and the frequencies $\{\omega_m\}$ are fixed but unknown parameters and $\{w_n\}$ are identical and independent Gaussian distributions with zero mean and variance σ_w^2. Since

$$\sin \Phi = (e^{j\Phi}-e^{-j\Phi})/2j$$

(1) can be rewritten in the vector-matrix form as

$$\underline{y} = (P \quad P') \begin{pmatrix} \underline{\alpha} \\ \underline{\alpha}' \end{pmatrix} + \underline{w} \tag{2}$$

where $\underline{y} = (y_0,...,y_{N-1})^t$.

$\underline{w} = (w_0,...,w_{N-1})^t$.

$\underline{\alpha} = (\alpha_1,..,\alpha_M)^t, \ \alpha_m=(a_m/2j)e^{j\theta m}$

$P_{NxM} = \{p_{nm}\}, \ p_{nm}=e^{jn\omega_m}$.

and the superscripts "t" and "'" denote matrix transpose and complex conjugate respectively. By leting $\underline{\beta} = \left(\frac{\underline{\alpha}}{\underline{\alpha}'}\right)$ and $S=(P \mid P')$ (2) can be further reduced to

$$\underline{y} = S\,\underline{\beta} + \underline{w} \tag{3}$$

Note that in the above formulation, each real sinusoid is decomposed into a pair of complex ones whose frequencies are equal in magnitude.

The ML method estimates the unknown parameters by maximizing the likelihood function which is a strictly increasing function of the quadratic form

$$Q_1(\underline{\omega},\underline{\beta}) = -(\underline{y}-S\underline{\beta})^t\,(\underline{y}-S\underline{\beta}) \tag{4}$$

where $\underline{\omega} = (\omega_1,\ldots,\omega_m)^t$.

By solving $\partial Q_1/\partial\underline{\beta}=0$ for $\underline{\beta}$, ($\underline{\beta} = (S^*S)^{-1}\,S^*\underline{y}$), and substituting it back into (4) followed by dropping the constant term $-\underline{y}^t\underline{y}$ we have

$$Q_2(\underline{\omega}) = (\underline{y}^t S)\;(S^*S)^{-1}(S^*\underline{y}) \tag{5}$$

or

$$Q_2(\underline{\omega}) = (\underline{y}^t P \quad \underline{y}^t P')\left(\begin{array}{cc|cc} P^* & P & P^* & P' \\ \hline P^t & P & P^t & P' \end{array}\right)^{-1}\left(\begin{array}{c} P^*\;\underline{y} \\ \hline P^t\;\;y \end{array}\right) \tag{6}$$

where the asterisk ("*") denotes conjugate transpose.

Now letting

$$\underline{d}_{Mx1} = \frac{1}{N} (P^*\underline{y})$$

$$A_{MxN} = \frac{1}{N} (P^*P)$$

$$B_{MxM} = \frac{1}{N} (P^*P')$$

and inverting the partioned matrix, we have

$$Q_2(\underline{\omega}) = 2N \{\underline{d}^* R \underline{d} + Re [\underline{d}^t T \underline{d}]\} \tag{7}$$

where $R = (A-B(A^t)^{-1}B')^{-1}$

and $T = -(A^t)^{-1} B'R$

It is worthwhile to point out that the component of $\underline{d}$, d_m is equal to

$$d(\omega) = \frac{1}{N} \sum_{n=0}^{N-1} y_n e^{-j\omega n} \tag{8}$$

evaluated at $\omega=\omega_m$ and the entries of A and B, a_{mn} and b_{mn}, are are equal to

$$\rho (\omega) = \frac{1}{N} \sum_{n=0}^{N-1} e^{-j\omega n} = \frac{\sin(N\omega/2)}{N \sin (\omega/2)} e^{-j (\frac{N-1}{2})\omega} \tag{9}$$

evalued at $\omega=\omega_m-\omega_n$ and $\omega=\omega_m+\omega_n$ respectively. So $Q_2(\underline{\omega})$ can be computed using $d(\omega)$ and $\rho(\omega)$ only and the frequency estimate $\hat{\underline{\omega}}$ is the $\underline{\omega}$ which maximizes this quantity. Also note that $d(\omega)$ is the DFT value of the observed signal $\{y_n\}$ at ω and $\rho(\omega)$ is the Dirichlet kernel associated with the rectangular window. In fact $\rho(\omega)$ also measures the interference between two spectral lines separated by ω. A plot of $\rho(\omega)$ for N=16, 32 and 64 is shown in Fig. 1.

Now let us consider some special cases. For the case of M=1 (a single real sinusoid), $Q_2(\underline{\omega})$ *can be written* explicitly as

$$Q_2(\omega_1) = 2N\{|(d(\omega_1)|^2 - \mathrm{Re}[d^2(\omega_1)\rho'(2\omega_1)]\}/(1-|\rho(2\omega_1)|^2) \quad (10)$$

Note that the interference term $\rho(2\omega_1)$ between the positive and negative spectral lines of the real sinusoid appears in the expression. In the limiting case $\omega_1=0$, $\rho=1$ and $Q_2=|d(\omega=0)|^2$.

Suppose $\omega_1 \leq \omega_2 \leq \omega_M$. If ω_1 is large enough so that the negative spectral lines associated with the real sinusoids do not interfere with the positive ones then $B \simeq 0$ and Q_2 reduces to

$$Q_2(\underline{\omega}) = 2Nd^*A^{-1}d \quad (11)$$

This is exactly the same formula as Eq. (51) of [2] for M

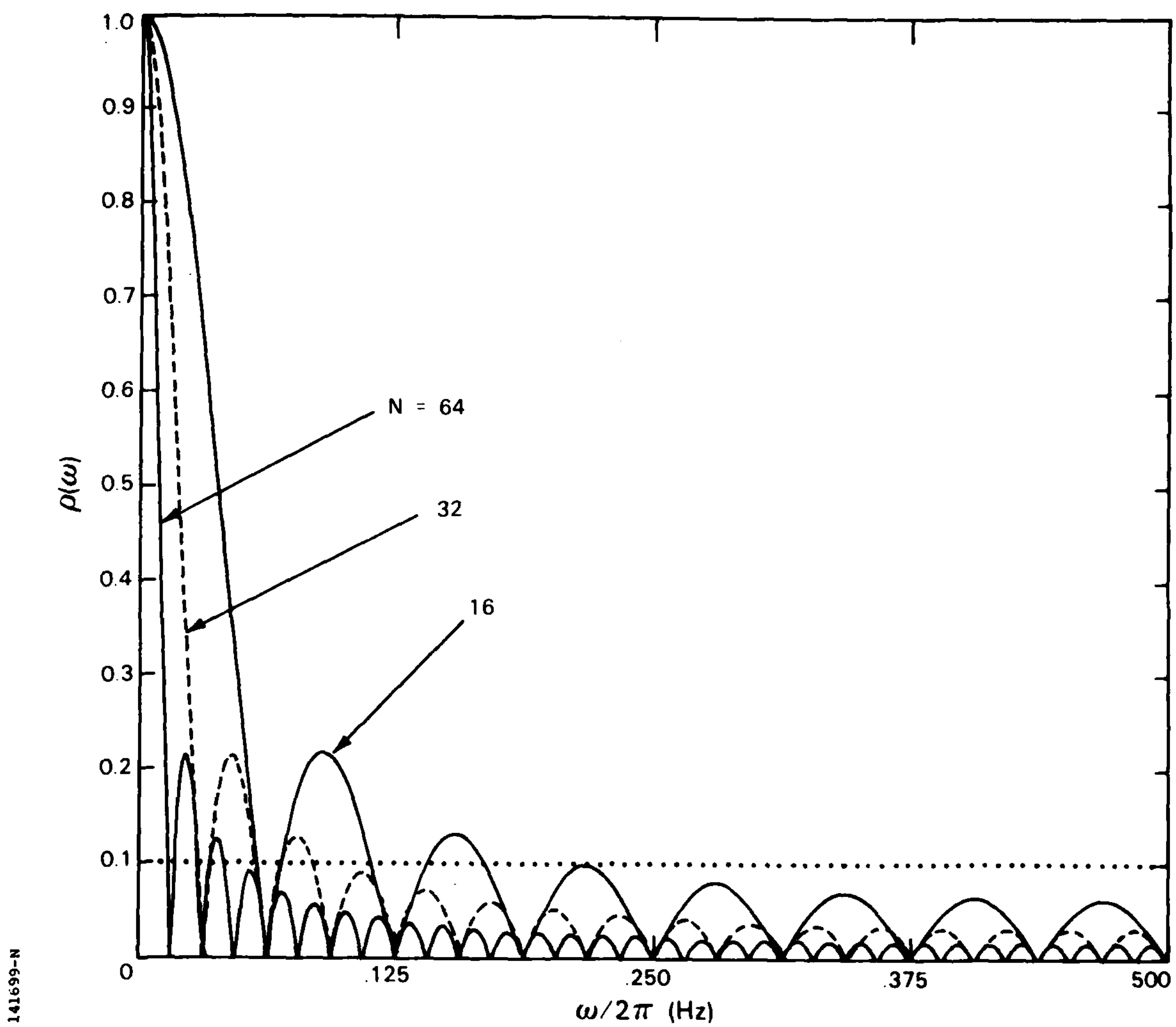

Fig. 1. $\rho(\omega)$ vs ω for N=16, 32 and 64.

complex sinusoids except the constant factor 2. Therefore, under this condition, the frequency estimates of M real sinusoids can be obtained by regarding them as M complex sinusoids with positive frequencies.

From (11) it is also clear that the corresponding formula for one and two complex sinusoids are, after dropping the factor 2,

$$Q_2(\omega_1) = N\left|d(\omega_1)\right|^2 \tag{12a}$$

and

$$\begin{aligned} Q_2'(\omega_1,\omega_2) &= N/(1-\left|\rho(\omega_2-\omega_1)\right|^2) \times \\ &\left\{\left|d(\omega_1)\right|^2+\left|d(\omega_2)\right|^2-2\ \mathrm{Re}(d(\omega_1)\ d'(\omega_2)\rho(\omega_2-\omega_1))\right\} \end{aligned} \tag{12b}$$

respectively. Note that if $\omega_2 = -\omega_1$ than (12b) reduces to (10) as expected.

B. Algorithm

To find the frequency estimate $\hat{\underline{\omega}}$, using either (10) or (11) we first perform, as proposed in [1-2], a finite search in M nested do-loops for the maximum of $Q_2(\underline{\omega})$ over a Ω^M lattice when $\Omega = \{2\pi k/K, k=0,\ldots K-1 \text{ and } K \geq N\}$. Empirically speaking, K=2N or 4N is enough as long as 1/K is comparable to the lowest frequency in the signal . The required $\{d(\omega), \omega\varepsilon\Omega)\}$

and $\{\rho(\omega),\ \omega\epsilon\Omega\}$ during the finite search can be fetched from the precomputed (using the FFT) and prestored tables. This course estimate $\underline{\tilde{\omega}}$ is then refined by using a single-step interpolation (i.e. one-iteration Newton's method) to give the final estimate

$$\underline{\hat{\omega}}=\underline{\tilde{\omega}}-H^{-1}\underline{g}$$

where $\underline{g}$ (gradient) = $\partial Q_2/\partial\underline{\omega}$ and H(Hessian) = $\partial\underline{g}^t/\partial\underline{\omega}$ are evaluted at $\underline{\tilde{\omega}}$ numerically (based on the central differences [5]) using values of $Q_2(\underline{\omega})$ already computed in the finite search. If $Q_2(\underline{\hat{\omega}})<Q_2(\underline{\tilde{\omega}})$ then $\underline{\tilde{\omega}}$ is taken as $\underline{\hat{\omega}}$. It is found that this simple interpolation can eliminate the lengthy iterations and the associated convergence problems and yet provide a satisfactory precision.

When M is large it will be difficult to carry out the search at one time because of the large search space Ω^M. Fortunately, in many cases, M' (<M) independent (non-interfering) and significant (detectable) intervals with length $\Omega' < \Omega$ can be identified from the DFT spectrum of the data, $\{|d(\omega)|,\omega\epsilon\Omega\}$, each of which contains one single or a small number of blurred spectral lines and these intervals can be treated individually in much smaller space using either (7) or (11) depending upon their distances from $\omega=0$.

III. SIMULATION RESULTS

Three examples are used here to test the estimation precision and computation efficiency of the proposed ML method. The first two have been utilized in other frequency estimators [6,7]. The computer simulation results for these two examples are compared with the published data. In the following the simulation conditions for each example are specified in terms of the observation model, the number of given samples, (N), the number of DFT values used in the search (K), the number of Monte-Carlo simulations (MC) and the signal-to-noise ratio which is defined as SNR =10 log $(1/2\sigma_w^2)$

Example 1: $y_n = \sin(.5\pi n + .5\pi) + w_n$, N=32

This example was used in [6] where the covariance and modified covariance methods of the "maximum entropy" (ME) spectral estimator were studied. The simulation conditions are MC=1000, SNR=37 dB and K=64. The best performances, in terms of estimation standard derviations, of the two ME methods as extracted from Fig. 2 of [6] are $.53 \times 10^{-4}$ Hz and $.47 \times 10^{-4}$ Hz. They are outperformed by the ML method where the standard deviation is $.36 \times 10^{-4}$ Hz.

Example 2: $y_n = \sin(.1\pi n) + w_n$, N=25

This example was used in [7] where the instrumental variable method (IVM) was proposed for estimating frquencies of real sinusoids. The simulation conditions are MC=200, K=64 and SNR =22.2, 12.2 and 7 dB. The comparison is given in Table I. Both Eq. (10) and Eq. (12b) are used. In the latter case, the real sinusoid is regarded as a pair of complex sinusoids and the actual frequency estimate is taken as the average of the magnitudes of the two estimated frequencies based on (12b).

From Table I, it is clear that the ML method is superior to the IVM and its estimation precision approaches to the Cramer-Rao bound (CRB)*[1]. Although the estimation precisions obtained by using (10) and (12b) are comparable, the required computing times are quite different. On a Amdhal 470 machine, each run takes $.4050 \times 10^{-2}$ seconds using (10) and .1026 seconds (25 times longer) using (12b).

Example 3: $y_n = \sin(.05\pi n + \pi/4) + w_n$

$N=16$

SNR=7,5 and 0 dB.

This example is intended to compare the performances of using Eq (10) and Eq. (12b) in the cases of low SNR's and strong interference between the positive and the negative spectral lines of the real sinusoid. The interference is 23% in this

*The CRB is a lower bound on the estimation variance of any unbiased estimator.

example. Table II gives the comparison. Undoubtedly, the proposed ML method (Eq. (10)) for frequency estimation of real sinusoid is much more precise than the one (Eq. (12b)) which is adapted from the ML method originally designed for complex data. From Table II also note that the estimation becomes biased at very low SNR, e.g., 0 dB.

TABLE I

Comparison of IVM, MLM with Eq. (10) and MLM with (Eq. 12b) for frequency estimation precision of a real sinusoid in noise. The true frequency is .05 Hz Each entry in column 2,3, and 4 means meand ±SD in units of Hz. See example 2.

Method SNR dB	IVM [6]	MLM Eq. (10)	MLM Eq. (12b)	$(CRB)^{1/2}$
22.2	$.5013 \times 10^{-1}$ $\pm 1.897 \times 10^{-3}$	$.4946 \times 10^{-1}$ $\pm .3827 \times 10^{-3}$	$.4958 \times 10^{-1}$ $\pm .3885 \times 10^{-3}$	$.3244 \times 10^{-3}$
12.2	$.5037 \times 10^{-1}$ $\pm 1.0904 \times 10^{-2}$	$.4956 \times 10^{-1}$ $\pm .1073 \times 10^{-2}$	$.4972 \times 10^{-1}$ $\pm .1089 \times 10^{-2}$	$.1026 \times 10^{-2}$
7.0	$.4819 \times 10^{-1}$ $\pm 3.0412 \times 10^{-2}$	$.4962 \times 10^{-1}$ $\pm .1941 \times 10^{-2}$	$.4974 \times 10^{-1}$ $\pm .1966 \times 10^{-2}$	$.1867 \times 10^{-2}$

TABLE II

Comparison of MLM with Eq. (10) and MLM with Eq. (12b) for frequency estimation precision of a real sinusoid. The true frequency is .025 Hz. See example 3.

SNR dB method	7	5	0
Eq. (10)	$.2313 \times 10^{-1}$ $\pm .5199 \times 10^{-2}$	$.2215 \times 10^{-1}$ $\pm .6044 \times 10^{-2}$	$.3251 \times 10^{-1}$ $\pm .5758 \times 10^{-1}$
Eq. (12b)	$.2599 \times 10^{-1}$ $\pm .1921 \times 10^{-1}$	$.3573 \times 10^{-1}$ $\pm .4230 \times 10^{-1}$	$.8304 \times 10^{-1}$ $\pm .8218 \times 10^{-1}$

IV. CONCLUSIONS

The ML method [1-2] for frequency estimation of complex sinusoids is extended to treat real data. This extension provides better estimation precision and computation efficiency than the conventional approaches. The detection problem(determining the number of sinusoids present in the data) which is not addressed here can be handled easily based on the M-ary generalized likelihood ratio test [8] using by-products of the ML estimator.

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